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Research problems

The purpose of the research problems section is the presentation of unsolved problems in discrete mathematics. Older problems are acceptable if they are not as widely known as they deserve. Problems should be submitted using the format as they appear in the journal and sent to

Professor Brian Alspach
Department of Mathematics and Statistics
Simon Fraser University
Burnaby, B.C.
Canada V5A 1S6.

Readers wishing to make comments dealing with technical matters about a problem that has appeared should write to the correspondent for that particular problem. Comments of a general nature about previous problems should be sent to Professor Alspach.

Problem 172. Posed by Mieczysław Borowiecki.

Correspondent: Mieczysław Borowiecki
Institute of Mathematics
Higher College of Engineering
Podgórna 50
65-246 Zielona Góra
Poland

Let G be a finite simple graph. Suppose that with each vertex v of G we associate $k \geqslant 1$ distinct colours. The graph G is said to be *k-choosable* if, no matter what colours are associated, we can always make a choice of one colour for each vertex, with distinct colour for adjacent vertices. The *choice number* of G, denoted by $\#(G)$, is equal to k if G is k-choosable but not $(k\text{-}1)$-choosable.

Problem. Prove or disprove

$$\#(G) \leqslant \eta(G),$$

where $\eta(G)$ is the Hadwiger number of G.

References

[1] P. Erdős, A.L. Rubin and H. Taylor, Choosability in graphs, in: Proc. West. Coast Conference on Combinatorics, Graph Theory, and Computing, Congr. Numer. 26 (1979) 125–157.
[2] P. Vaderlind, Choosability in graphs: Some results and open problems. A survey, in: M. Borowiecki and Z. Skupień, eds., Proc. 7th Regional Scientific Session of Mathematicians, Kalsk, September 1988, Graphs, Hypergraphs and Matroids III, WSI Publ., Zielona Góra, 1989, 157–163.

Problem 173. Posed by Miroslav Fiedler.
Correspondent: Miroslav Fiedler
Math. Inst. Acad.
Zitna 25
11567 Praha 1
Czech Rupublic

Problem. Prove (or disprove) that a fully indecomposable [1] orthogonal matrix of order n has at least $4(n-1)$ nonzero entries.

Comment: True for $n=2$, 3, 4, and there exist such matrices with $4(n-1)$ nonzero entries for each $n \geqslant 2$.

Added in proof. Since the problem was proposed, it hat has subsequently been solved (in the affirmative) by L.B. Beasly, R.A. Brualdi and B.L. Shader.

References

[1] H. Minc, Permanents (Addison-Wesley, Reading, MA, 1978) (Russian Ed., Mir, Moskva 1982).

Problem 174. Posed by Antoni Marczyk and Zdzisław Skupień.
Correspondent: Antoni Marczyk
Institute of Mathematics
Academy of Mining and Metallurgy
Al. Mickiewicza 30
30-059 Kraków
Poland

Consider a connected n-vertex graph H_n ($n \geqslant 6$) which is the complete graph K_{n-3} together with three independent hanging edges. Denote by M_n ($n \geqslant 7$) the join of K_1 and H_{n-1} and by G_9 the join of K_2 and the union $K_1 \cup H_6$.

It is easy to show that the graphs M_n and G_9 are maximally nonhamiltonian tough graphs of order n and size $\binom{n-3}{2}+6$.

Recently Marczyk and Skupień [1] have proved that the maximum size of a tough nonhamiltonian n-vertex graph is $6+(n-3)(n-4)/2$ where $n \geqslant 7$. The corresponding maximum graphs are M_n for each $n \geqslant 7$ and, additionally, G_9 for $n=9$.

Each of these graphs has one or two total vertices (of degree $n-1$). Observe that if G is a maximally nonhamiltonian graph then G is homogeneously traceable if and only if G has no total vertex. Consider the following problem.

Problem. Find maximum homogeneously traceable nonhamiltonian graphs on n vertices for $n \geqslant 11$. In particular find the size of such graphs.

Exponentially many minimum nonhamiltonian homogeneously traceable graphs (or digraphs) are constructed in [2].

References

[1] A. Marczyk and Z. Skupień, Maximum nonhamiltonian tough graphs, Discrete Math. 96 (1991) 213–220.

[2] Z. Skupień, Exponential constructions of some nonhamiltonian minima, in: J. Nešetril and M. Fiedler, eds., Fourth Czechoslovakian Symposium on Combinatorics, Graphs and Complexity, Proc. Symp., Prachatice 1990, Ann. Discrete Math. 51 (Elsevier, Amsterdam, 1992) 321–328.

Problem 175. Posed by Günter Schaar.

Correspondent: Günter Schaar
Bergakademie Freiberg
Sektion Mathematik
B.-v.-Cotta-Str.2
O-9200 Freiberg
Germany

A block H is said to be minimal if and only if for any edge e the subgraph $H-e$ is not a block. For $p=3, 4, 5$ the following statement is valid.

If H is a minimal block with circumference at least p then there exists a Hamiltonian cycle in H^2 containing at least p edges of H.

Problem. Does this statement remain valid for $p \geqslant 6$?